REDUCING QUADRATIC FRACTIONS

❑ INTRODUCTION

Now that we're proficient at factoring, we can knock some ugly fractions down to size. Let's review a few reducing problems to prepare for this chapter.

First Fraction: Let's reduce the fraction $\frac{39}{52}$. By prime factoring the top and bottom, we get $\frac{39}{52} = \frac{3\cdot 13}{2\cdot 2\cdot 13} = \frac{3\cdot \cancel{13}^{1}}{2\cdot 2\cdot \cancel{13}_{1}} = \boldsymbol{\frac{3}{4}}$. Once the top and bottom have been factored (i.e., written as a product), we can divide out (or cancel) the common factor of 13.

Second Fraction: Applying the same logic to the fraction $\frac{ax}{a^2+7a}$, we can reduce it by writing $\frac{ax}{a^2+7a} = \frac{ax}{a(a+7)} = \frac{\cancel{a}x}{\cancel{a}(a+7)} = \boldsymbol{\frac{x}{a+7}}$.

And a Third Fraction: $$\frac{LT-QT}{AL-AQ} = \frac{\overbrace{T(L-Q)}^{\text{top factored}}}{\underbrace{A(L-Q)}_{\text{bottom factored}}} = \frac{T\cancel{(L-Q)}^{1}}{A\cancel{(L-Q)}_{1}} = \boldsymbol{\frac{T}{A}}$$

And don't forget that when something is factored, it consists of a single term (where *multiplication* is the *final operation*). Thus, $a(b-c)$ is factored, but $nx + ny$ is <u>not</u> factored. Most importantly, notice that in each fraction we have canceled *factors* (things multiplied), <u>not</u> *terms* (things added).

Homework

1. Verify that the fraction $\frac{ab+ac}{ad}$ reduces to $\frac{b+c}{d}$ by letting $a = 2$, $b = 3$, $c = 4$, and $d = 6$ in both fractions.

2. Prove that the fraction $\frac{x+z}{yz}$ does <u>not</u> reduce to $\frac{x}{y}$ by choosing numerical values for x, y, and z. Also, prove to yourself that $\frac{x+z}{yz}$ does <u>not</u> reduce to $\frac{x+1}{y}$, either.

3. First, consider the fraction $\frac{x^2+2}{x^2+3}$. By letting $x = 1$, prove that the fraction does <u>not</u> reduce to $\frac{2}{3}$.
   Second, look at the fraction $\frac{2x^2}{3x^2}$. Note that this fraction <u>is</u> reducible and that it reduces to $\frac{2}{3}$.
   Third, explain why the first fraction is <u>not</u> reducible while the second fraction <u>is</u> reducible.

❑ *REDUCING QUADRATIC FRACTIONS*

Whether we're reducing $\frac{39}{52}$ (like the one at the top of the chapter) or something like what you're about to see, the process is the same:

1) Factor the top and factor the bottom.
2) Divide out any common factors.

See *Factoring Quadratics – The Real Deal* for a reminder on factoring quadratics.

<u>EXAMPLES:</u> **Reduce each fraction to lowest terms:**

A. $\dfrac{x^2+5x+6}{x^2-4} = \dfrac{(x+3)(x+2)}{(x+2)(x-2)} = \dfrac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \boldsymbol{\dfrac{x+3}{x-2}}$

B. $\dfrac{6n^2+5n-21}{3n^2+22n+35} = \dfrac{(3n+7)(2n-3)}{(3n+7)(n+5)} = \dfrac{\cancel{(3n+7)}(2n-3)}{\cancel{(3n+7)}(n+5)} = \boldsymbol{\dfrac{2n-3}{n+5}}$

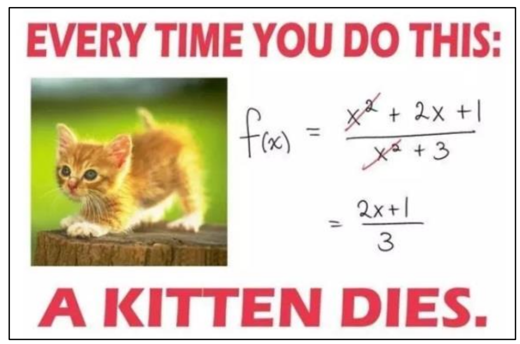
C. $\dfrac{x+5}{2x^2+7x-15} = \dfrac{x+5}{(2x-3)(x+5)} = \dfrac{\cancel{x+5}}{(2x-3)\cancel{(x+5)}} = \boldsymbol{\dfrac{1}{2x-3}}$

D. $\dfrac{3w^2-2w-5}{3w-5} = \dfrac{(3w-5)(w+1)}{3w-5} = \dfrac{\cancel{(3w-5)}(w+1)}{\cancel{3w-5}} = \boldsymbol{w+1}$

E. $\dfrac{a^2-25}{a^2+5a+6} = \dfrac{(a+5)(a-5)}{(a+3)(a+2)}$,

which contains no common factors.

Therefore, this fraction is **not reducible**.



Homework

4. Reduce each fraction to lowest terms:

a. $\dfrac{n^2-9}{n^2+6n+9}$ b. $\dfrac{4a^2+4a+1}{2a^2+5a+2}$ c. $\dfrac{c+7}{c^2-49}$

d. $\dfrac{4w^2-9}{2w-3}$ e. $\dfrac{6x^2+11x-7}{2x^2+17x-9}$ f. $\dfrac{h^2+3h+2}{h^2-3h+2}$

g. $\dfrac{3k^2 - 17k + 1}{3k^2 - 17k + 1}$ h. $\dfrac{100 - w^2}{w^2 + 10w}$ i. $\dfrac{10x^2 + 11x - 6}{5x^2 - 12x + 4}$

j. $\dfrac{x^2 + 7x + 10}{x^2 + 3x + 2}$ k. $\dfrac{n^2 - 9}{n^2 + 4n + 3}$ l. $\dfrac{y - 7}{y^2 - 14y + 49}$

m. $\dfrac{w^2 - 81}{w + 9}$ n. $\dfrac{m^2 + 10m + 25}{m^2 - 25}$ o. $\dfrac{x^2 + 2x + 1}{x^2 - 4}$

p. $\dfrac{6a^2 + 13a - 5}{9a^2 + 12a - 5}$ q. $\dfrac{k^2 - 6k + 7}{7 - 6k + k^2}$ r. $\dfrac{16u^2 + 34u - 15}{2u^2 + 3u - 5}$

Review Problems

5. Reduce: $\dfrac{6a^2 - 5a - 21}{3a^2 - 4a - 7}$

6. Reduce: $\dfrac{x^2 - 9}{x^2 - 4}$

7. Reduce: $\dfrac{10a^2 + 29a - 21}{5a^2 - 38a + 21}$

8. Reduce: $\dfrac{x - 3}{x^2 - 9}$

9. Reduce: $\dfrac{n^2 + 14n + 49}{(n + 7)^2}$

Solutions

1. Note that this does not constitute a complete proof.

2. Be sure you don't let the y or the z be zero. How come?

3. Letting $x = 1$, $\frac{x^2+2}{x^2+3} = \frac{1^2+2}{1^2+3} = \frac{1+2}{1+3} = \frac{3}{4}$, which is not $\frac{2}{3}$.
The first fraction has two terms top and bottom; you cannot cancel terms. The second fraction has one term top and bottom (the final operation is multiplication); you are allowed to cancel factors.

4. a. $\frac{n-3}{n+3}$ b. $\frac{2a+1}{a+2}$ c. $\frac{1}{c-7}$ d. $2w + 3$

e. $\frac{3x+7}{x+9}$ f. Not reducible g. 1 h. $\frac{10-w}{w}$

i. $\frac{2x+3}{x-2}$ j. $\frac{x+5}{x+1}$ k. $\frac{n-3}{n+1}$ l. $\frac{1}{y-7}$

m. $w - 9$ n. $\frac{m+5}{m-5}$ o. Not Reducible p. $\frac{2a+5}{3a+5}$

q. 1 r. $\frac{8u-3}{u-1}$

5. $\frac{2a+3}{a+1}$ **6.** Not reducible **7.** $\frac{2a+7}{a-7}$

8. $\frac{1}{x+3}$ **9.** 1

❑ *TO ∞ AND BEYOND*

A. Consider the fraction

$$\frac{3}{x-7}$$

Let's choose a number for x. We see that if $x = 4$, for instance, then the value of the fraction is

$$\frac{3}{4-7} = \frac{3}{-3} = -1$$

And x can also take on the value 0, since $\frac{3}{\mathbf{0}-7} = \frac{3}{-7} = -\frac{3}{7}$. In fact, x can be any number at all . . . with one exception. What is that number?

B. Consider the fraction

$$\frac{x^2-4}{x^2-11x+30}$$

Find the two values of x which must never be used in this fraction.

"Education is what survives when what has been learned has been forgotten."

– **B. F. Skinner** (1904 - 1990)